

# Effects of hadronic loops on the direct CP violation of $B_c$

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It is well known that the final state interaction plays an important role in the decays of  $B$ -meson. The contribution of the final state interaction which is supposed to be long-distance effects, to the concerned processes can interfere with that of the short-distance effects produced via the tree and/or loop diagrams at quark-gluon level. The interference may provide a source for the direct CP violation  $\mathcal{A}_{CP}$  in the process  $B_c^+ \rightarrow D^0 \pi^+$ . We find that a typical value of  $\mathcal{A}_{CP}$  when the final state interaction effect is taken into account can be about  $-22\%$  which is different from that without the final state interaction effect. Therefore, when we extract information on CP violation from the data which will be available at LHCb and the new experiments in  $B$ -factories, the contribution from the final state interaction must be included. This study may be crucial for searching new physics in the future.

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## I. INTRODUCTION

One of the most intriguing goals in the high energy physics is to look for new physics beyond the Standard Model (SM) via heavy hadron production and decay processes. The reason is that new physics which generally has a higher energy scale may be observed at the processes involving heavy flavors. Among all the possible quantities which are experimentally measurable, CP violation provides a more sensitive window to the new physics effects. Direct CP violation at  $B$ -physics has been observed by the Babar and Belle collaborations [1, 2], which is indeed a great success after confirmation of non-zero  $\epsilon'/\epsilon$  at K-systems. Another promising place to study CP violation is the meson  $B_c$ , which is composed of different heavy flavors.

Since the CDF Collaboration observed  $B_c$  meson in the semileptonic decay  $B_c \rightarrow J/\psi + l + \nu$  [3], studies on  $B_c$  have drawn great interests from both theorists and experimentalists of high energy physics. Decays of  $B_c$  can be realized via  $b$ -decay,  $\bar{c}$ -decay and annihilation of  $b$  and  $\bar{c}$  [4]. Many theoretical works have been dedicated to study the decays of  $B_c$  [5, 6, 7, 8]. A relatively complete discussion about its spectrum, production and decays was presented in a review [9]. Because of the specific characteristics of its decay modes, the direct CP violation is an important observable which may provide valuable information towards the mechanism governing the transition and probably unveils a trace to the new physics beyond the SM.

In this work, we are just looking for a new source for the direct CP violation in  $Bc$  decays. The direct CP violation is caused in general, by an interference among at least two channels which have the same final state, but different weak and strong phases. The CP quantity  $\mathcal{A}_{CP}$  is proportional to

$$\mathcal{A}_{CP} = \frac{2|A_1||A_2|\sin(\theta_1 - \theta_2)\sin(\alpha_1 - \alpha_2)}{|A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\theta_1 - \theta_2)\cos(\alpha_1 - \alpha_2)},$$

where  $A_1, A_2$  are the amplitudes of the two distinct channels and  $\theta_1, \theta_2, \alpha_1, \alpha_2$  are their strong phases and weak phases respectively.

These phase differences coming from either quark level or hadron level. At the quark level the strong phase difference usually occurs via the absorptive part of the loops involved in the calculation. The strong phase may also occur at the hadron level. As a matter of fact, it is well known in the kaon system. When one studies the direct CP violation, i.e.  $\epsilon'/\epsilon$ , the phase shifts in the  $\pi\pi$  scattering provide the strong phase which are necessary to result in CP violation. But recently most of the works to study direct CP violation concentrate on the strong phase induced by the absorptive part of the loops. Especially, the strong phase is coming from the absorptive part of the penguin diagram(s) which contribute along with the tree diagram to the amplitude. In that case the CP violation is induced by the interference between the contribution of the tree diagram and that of penguin.

The total width is related to  $|A_1 + A_2|^2$ . If one of the amplitudes is much smaller than the other one, the width should be only depend on the larger one, say  $A_1$ , thus one can ignore the smaller one when he is calculating the decay width. However, even though  $|A_2| \ll |A_1|$ , the numerator of  $A_{CP}$  is proportional to their product, so one cannot ignore the smaller contribution, otherwise he would get null CP asymmetry.

In that case, obviously the contribution from the penguin diagram is much smaller than that from the tree diagram, so that if only the decay width is needed, one can completely ignore the contribution of the penguin. However, for evaluating the CP violation, he by no means can dismiss the penguin contribution.

In the SM, the weak phase originates from the Cabibbo-Kabayashi-Maskawa matrix, and the strong phase is induced by the absorptive part of loops. At the quark-gluon level which is responsible for the short distance effects, the strong phases may originate from the absorptive part of loops, for example, the penguin diagrams. On the other aspect, the final state interaction (FSI) plays an important role in  $B$ -physics, as fully discussed in literature [10]. At the short distance, the direct CP violation usually is caused by an interference between the tree-level contribution and the loop-induced one because they have different weak and strong phases (in fact the tree diagrams do not contribute a strong phase). Therefore an interference of the long-distance contribution with the short-distance ones may change the theoretical prediction on the CP violation. In fact, the FSI effect is extensively applied to the discussion of the CP violation of  $B$  and  $D$  decays [11, 12, 13].

Indeed, by the quantum field theory, the lagrangian can be a combination of various pieces and each of them corresponds to different processes. For our transition matrix element  $M =_{out} \langle f|i \rangle_{in}$ , one has

$$M = \langle f|L_{PQCD}^{(1)} + T[L_{had}L_{PQCD}^{(2)}]|i\rangle,$$

where  $L_{PQCD}^{(1),(2)}$  corresponds to the lagrangian which includes QCD and weak or electromagnetic interactions, the superscripts (1) and (2) denote the lagrangians which can lead to different final states, whereas  $L_{had}$  is the lagrangian at hadron level. Then we further write the matrix element as

$$M = \langle f|L_{PQCD}^{(1)}|i\rangle + \sum_n \langle f|L_{had}|n\rangle \langle n|L_{PQCD}^{(2)}|i\rangle, \quad (1)$$

where the intermediate states  $|n\rangle$  are a complete set of hadrons with proper quantum numbers and the matrix element  $\langle f|L_{had}|n\rangle$  is just the hadronic scattering process and corresponds to the hadronic loops in our work.

Generally, the long-distance effects due to the FSI refer to the re-scattering of the intermediate hadrons which emerge at the direct decays, into the concerned final state and it is depicted by the term  $\langle n|L_{PQCD}^{(2)}|i\rangle$ . In these channels with the intermediate hadronic intermediate states may have different weak phases from that of the short-distance production channel occurring at quark-gluon-level. In the re-scattering processes, phase shifts exist due to strong interaction and thus can offer strong phases. And an extra strong phase which is definitely different from that induced by the quark-level loops, occurs from the hadron re-scattering processes  $\langle f|L_{had}|n\rangle$ .

Since the loop contribution is suppressed by the loop integration, generally the second term of the above equation is smaller than the first one which we may call it as the "tree" level contribution( but maybe not the tree diagram in the common sense).

The traditional PQCD calculation only takes care of the first term  $\langle f|L_{PQCD}^{(1)}|i\rangle$  and  $\langle n|L_{PQCD}^{(2)}|i\rangle$  in the second one, but leaves the part  $\langle f|L_{had}|n\rangle$  to be dealt with in other theories, for example, the chiral lagrangian and etc. at the hadron level. This picture is clearly depicted in Cheng's paper [10]. It indicates that unless the "tree" contribution (i.e. the first term) is suppressed by some mechanism, the first term corresponds to the direct process, so that is always dominating for the total amplitude. If we only need to consider the total decay width, the second term may contribute a smaller portion (sometimes it might be enhanced by some mechanism, but generally is much smaller). However, as we deal with the CP violation and need at least two different channels, their interference enforces us not to abandon this term even though it might be much smaller than the first one. Indeed, one may argue that the loop diagram, such as penguin can also contribute a strong phase and interfere with the tree contribution to result in a direct CP violation, the loop contribution may have a similar order as we considered here and possibly even smaller. At least as we state above, we are looking for a possible source of CP violation in  $B_c$  decays, i.e. the hadronic loops may contribute strong phases and cause sizable effects on CP violation as our numerical results given in the paper indicate.

Therefore we would say that the PQCD framework works well, but we instead are looking for a supposed-to-be smaller effect which can result in observable CP violation. If there is a small double-counting (could be), that is because the wavefunction adopted in the calculation is not well defined. In fact because  $|n\rangle$  generally are not the same as  $|f\rangle$ , the double-counting does not appear.

At present the direct CP violation in  $B_c$  decays due to short distance contribution has been studied by many authors [14, 15, 16, 17, 18]. But so far, the studies of the FSI effects on the direct CP Violation of  $B_c$  are absent. In

this work, by taking into account the long distance contribution caused by the FSI, we would re-evaluate the direct CP violation in the decays of  $B_c$ . Namely, we add a new contribution to the amplitude which has different strong and weak phases from that of short distance contributions which were calculated by many authors, thus their interference will significantly change the value of CP asymmetry in decays of  $B_c$ .

Indeed, before one can claim a discovery of new physics, he must exhaust all possibilities which the SM can provide. Therefore this work is also serving for the purpose and to determine if the FSI can result in a sizable contribution to the direct CP violation of  $B_c$ .

We choose the channel  $B_c^+ \rightarrow D^0\pi^+$  which should be one of the dominant decay modes of  $B_c$ . In this channel, there exist hadronic intermediate states which are mainly composed of  $D^{(*)+}$  and  $J/\psi$ .

This paper is organized as follow. We present the formulation about  $B_c^+ \rightarrow D^{(*)+}J/\psi \rightarrow D^0\pi^+$  in II. Then we present our numerical results. The last section is a short conclusion and discussion.

## II. FORMULATION

The effective Hamiltonian related to  $B_c$  decays is [19]

$$\begin{aligned} \mathcal{H}_{eff} = & \frac{G_F}{\sqrt{2}} V_{cb} V_{cd}^\dagger \left\{ \mathcal{C}_1^b(\mu) (\bar{c}b)_{V-A} (\bar{d}c)_{V-A} + \mathcal{C}_2^b(\mu) (\bar{d}b)_{V-A} (\bar{c}c)_{V-A} \right\} \\ & + \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^\dagger \left\{ \mathcal{C}_1^b(\mu) (\bar{u}b)_{V-A} (\bar{d}u)_{V-A} + \mathcal{C}_2^b(\mu) (\bar{d}b)_{V-A} (\bar{u}u)_{V-A} \right\}, \end{aligned} \quad (2)$$

where the subscript  $V-A$  denotes the left-chiral current  $\gamma^\mu(1-\gamma^5)$ .  $\mathcal{C}_{1,2}^b(\mu)$  denote the Wilson coefficients.

Firstly we calculate the transition amplitude of  $B_c \rightarrow D^{(*)+}J/\psi$  at the quark level and the hadronization would be described by a few phenomenological parameters. The definitions of the relevant hadronic matrix elements are

$$\langle 0 | \mathcal{J}_\mu | \mathcal{P}(k) \rangle = -i f_{\mathcal{P}} k_\mu, \quad (3)$$

$$\langle 0 | \mathcal{J}_\mu | \mathcal{V}(k, \epsilon) \rangle = f_{\mathcal{V}} \epsilon_\mu m_{\mathcal{V}}, \quad (4)$$

where  $f_{\mathcal{P}}$  and  $f_{\mathcal{V}}$  respectively stand for leptonic decay constants of pseudoscalar and vector mesons.  $k_\mu$  is the four-momentum of the concerned hadron and  $\epsilon_\mu$  denotes the polarization of the vector meson. One has  $\mathcal{J}_\mu = \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2$ .

In addition, the hadronic matrix elements of  $B_c$  transiting into two mesons can be expressed in terms of a few form factors as [19]

$$\langle \mathcal{P}(k_2) | \mathcal{J}_\mu | B_c(k_1) \rangle = P_\mu f_+(Q^2) + Q_\mu f_-(Q^2), \quad (5)$$

$$\begin{aligned} \frac{1}{i} \langle \mathcal{V}(k_2, \epsilon) | \mathcal{J}_\mu | B_c(k_1) \rangle = & \frac{\epsilon_\nu^*}{m_1 + m_2} \left\{ i \varepsilon^{\mu\nu\alpha\beta} P_\alpha Q_\beta F_V(Q^2) - g^{\mu\nu} (P \cdot Q) F_0^A(Q^2) \right. \\ & \left. + P^\mu P^\nu F_+^A(Q^2) + Q^\mu P^\nu F_-^A(Q^2) \right\} \end{aligned} \quad (6)$$

with  $P_\mu = (k_1 + k_2)_\mu$  and  $Q_\mu = (k_1 - k_2)_\mu$ . With the above formulas, we obtain

$$\begin{aligned} & \mathcal{M}[B_c^+(p) \rightarrow D^+(p_1) J/\psi(p_2)] \\ = & \frac{i G_F}{\sqrt{2}} V_{cb} V_{cd}^\dagger \left\{ a_1 f_D \frac{p_{1\sigma}}{m_{B_c} + m_\psi} \left[ -g^{\sigma\lambda} (p + p_2) \cdot (p - p_2) F_0^A(q_1^2) + (p + p_2)^\sigma (p + p_2)^\lambda F_+^A(q_1^2) \right. \right. \\ & \left. \left. + (p - p_2)^\sigma (p + p_2)^\lambda F_-^A(q_1^2) \right] + a_2 f_\psi m_\psi \left[ (p + p_1)^\lambda f_+(q_2^2) + (p - p_1)^\lambda f_-(q_2^2) \right] \right\}, \end{aligned} \quad (7)$$

and

$$\begin{aligned} & \mathcal{M}[B_c^+(p) \rightarrow D^{*+}(p_1) J/\psi(p_2)] \\ = & \frac{i G_F}{\sqrt{2}} V_{cb} V_{cd}^\dagger \left\{ a_1 f_{D^*} m_{D^*} \frac{i}{m_{B_c} + m_\psi} \left[ i \varepsilon^{\sigma\omega\tau\delta} (p + p_2)_\tau (p - p_2)_\delta F_V(q_1^2) \right. \right. \\ & - g^{\sigma\omega} (p + p_2) \cdot (p - p_2) F_0^A(q_1^2) + (p + p_2)^\sigma (p + p_2)^\omega F_+^A(q_1^2) \\ & \left. \left. + (p - p_2)^\sigma (p + p_2)^\omega F_-^A(q_1^2) \right] + a_2 f_\psi m_\psi \frac{i}{m_{B_c} + m_{D^*}} \left[ i \varepsilon^{\omega\sigma\tau\delta} (p + p_1)_\tau (p - p_1)_\delta F_V(q_2^2) \right. \right. \\ & \left. \left. - g^{\omega\sigma} (p + p_1) \cdot (p - p_1) F_0^A(q_2^2) + (p + p_1)^\omega (p + p_1)^\sigma F_+^A(q_2^2) + (p - p_1)^\omega (p + p_1)^\sigma F_-^A(q_2^2) \right] \right\} \end{aligned} \quad (8)$$

with  $q_1 = p - p_2$  and  $q_2 = p - p_1$ . The values of  $a_{1,2}$  will be given in next subsection.

### A. Absorptive part of hadronic loop for $B_c^+ \rightarrow D^{(*)+} J/\psi \rightarrow D^0 \pi^+$

Now let us turn to evaluate the contribution from the long-distance effects which occur at the hadron level. The diagrams shown in Fig. 1 depict sequent processes  $B_c^+ \rightarrow D^{(*)+} J/\psi \rightarrow D^0 \pi^+$ .

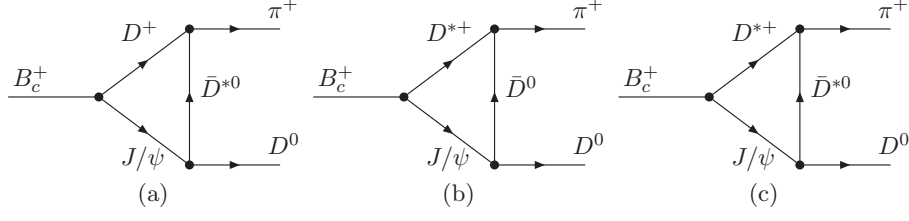


FIG. 1: The final state interaction contributions to  $B_c^+ \rightarrow D^0 \pi^+$ .

The effective Lagrangian at the hadronic level is suggested to be in the following forms as [20]

$$\mathcal{L}_{\mathcal{D}^* \mathcal{D} \pi} = ig_{\mathcal{D}^* \mathcal{D} \pi} (\mathcal{D}_\mu^* \partial^\mu \pi \bar{\mathcal{D}} - \mathcal{D} \partial^\mu \pi \bar{\mathcal{D}}_\mu^*), \quad (9)$$

$$\mathcal{L}_{\mathcal{D}^* \mathcal{D}^* \pi} = -g_{\mathcal{D}^* \mathcal{D}^* \pi} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \mathcal{D}_\nu^* \pi \partial_\alpha \bar{\mathcal{D}}_\beta^*, \quad (10)$$

$$\mathcal{L}_{\psi \mathcal{D} \mathcal{D}} = ig_{\psi \mathcal{D} \mathcal{D}} \psi_\mu (\partial^\mu \mathcal{D} \bar{\mathcal{D}} - \mathcal{D} \partial^\mu \bar{\mathcal{D}}), \quad (11)$$

$$\mathcal{L}_{\psi \mathcal{D}^* \mathcal{D}} = -g_{\psi \mathcal{D}^* \mathcal{D}} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \psi_\nu (\partial_\alpha \mathcal{D}_\beta^* \bar{\mathcal{D}} + \mathcal{D} \partial_\alpha \bar{\mathcal{D}}_\beta^*) \quad (12)$$

with  $\pi = \boldsymbol{\tau} \cdot \boldsymbol{\pi}$ , where fields  $\mathcal{D}^{(*)}$  and  $\bar{\mathcal{D}}^{(*)}$  are defined as  $\mathcal{D}^{(*)} = (D^{(*)0}, D^{(*)+})$  and  $\bar{\mathcal{D}}^{(*)T} = (\bar{D}^{(*)0}, \bar{D}^{(*)-})$ .

The process shown in Fig. 1 (a) is  $B_c^+ \rightarrow D^+(p_1) J/\psi(p_2) \rightarrow \pi^+(p_3) D^0(p_4)$  where  $D^{*0}$  is exchanged at t-channel, and its amplitude reads

$$\begin{aligned} Abs^{(a)} &= \frac{1}{2} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(m_{B_c} - p_1 - p_2) \mathcal{M}[B_c^+(p) \rightarrow D^+(p_1) J/\psi(p_2)] \\ &\times [-g_{D^* \mathcal{D} \pi} i p_3^\xi] [-ig_{J/\psi D^* D} \varepsilon^{\mu\nu\alpha\beta} (-ip_{2\mu}) i q_\alpha] (-g_{\lambda\nu} + \frac{p_{2\lambda} p_{2\nu}}{m_\psi^2}) \\ &\times (-g_{\beta\xi} + \frac{q_\beta q_\xi}{m_{D^*}^2}) \frac{i}{q^2 - m_{D^*}^2} \mathcal{F}^2[q^2, m_{D^*}^2]. \end{aligned} \quad (13)$$

Obviously, the conservation of angular momentum demands the contribution from Fig. 1 (a) to be zero.

The amplitude corresponding to the process of  $B_c^+ \rightarrow D^{*+}(p_1) J/\psi(p_2) \rightarrow \pi^+(p_3) D^0(p_4)$  where  $D^0$  is exchanged at t-channel reads as

$$\begin{aligned} Abs^{(b)} &= \frac{1}{2} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(m_{B_c} - p_1 - p_2) \mathcal{M}[B_c^+(p) \rightarrow D^{*+}(p_1) J/\psi(p_2)] \\ &\times [-g_{D^* \mathcal{D} \pi} (-ip_3^\xi)] [-g_{\psi \mathcal{D} D} (ip_4 - iq)^\mu] (-g_{\sigma\xi} + \frac{p_{1\sigma} p_{1\xi}}{m_{D^*}^2}) \\ &\times (-g_{\omega\mu} + \frac{p_{2\omega} p_{2\mu}}{m_\psi^2}) \frac{i}{q^2 - m_D^2} \mathcal{F}^2[q^2, m_D^2]. \end{aligned} \quad (14)$$

For Fig. 1 (c),  $B_c^+ \rightarrow D^{*+}(p_1) J/\psi(p_2) \rightarrow \pi^+(p_3) D^0(p_4)$  where  $D^{*0}$  is exchanged at t-channel, the amplitude is

$$\begin{aligned} Abs^{(c)} &= \frac{1}{2} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(m_{B_c} - p_1 - p_2) \mathcal{M}[B_c^+(p) \rightarrow D^{*+}(p_1) J/\psi(p_2)] \\ &\times [-ig_{D^* \mathcal{D}^* \pi} \varepsilon^{\mu\nu\alpha\beta} (-ip_{1\mu}) (-iq_\alpha)] [-ig_{J/\psi D^* D} \varepsilon^{\xi\lambda\kappa\rho} (-ip_{2\xi}) i q_\kappa] \\ &\times (-g_{\beta\rho} + \frac{q_\beta q_\rho}{m_{D^*}^2}) (-g_{\sigma\nu} + \frac{p_{1\sigma} p_{1\nu}}{m_{D^*}^2}) (-g_{\omega\lambda} + \frac{p_{2\omega} p_{2\lambda}}{m_\psi^2}) \frac{i}{q^2 - m_{D^*}^2} \mathcal{F}^2[q^2, m_{D^*}^2]. \end{aligned} \quad (15)$$

In the above amplitudes,  $q = p_3 - p_1$ , and  $\mathcal{F}(q^2, m_i)$  etc. denote the form factors which compensate the off-shell effects of mesons at the effective vertices and may be described by the possible pole structures [10]

$$\mathcal{F}(q^2, m_i) = \left( \frac{\Lambda^2 - m_i^2}{\Lambda^2 - q^2} \right)^n, \quad (16)$$

where  $\Lambda$  is a phenomenological parameter to be determined. As  $q^2 \rightarrow 0$  the form factor becomes a number. If  $\Lambda \gg m_i$ , it becomes a unity. As  $q^2 \rightarrow \infty$ , the form factor approaches to zero. It reflects the fact that as the distance between the mesons becomes very small, their inner structures would overlap and the whole picture of hadron interaction breaks down. Hence the form factor vanishes at large  $q^2$  and effectively plays a role to cut off the ultraviolet divergence. The expression of  $\Lambda$  is suggested to be [10]

$$\Lambda(m_i) = m_i + \alpha \Lambda_{QCD}, \quad (17)$$

where  $m_i$  denotes the mass of the exchanged meson and  $\alpha$  is a phenomenological parameter. In this work, we adopt the dipole form factor  $\mathcal{F}(q^2, m_i) = (\Lambda^2 - m_i^2)^2 / (\Lambda^2 - q^2)^2$ .

### B. The dispersive part of $B_c^+ \rightarrow D^{(*)+} J/\psi \rightarrow D^0 \pi^+$

In the above subsection, the absorptive part of the triangle diagram to the amplitude of the sequent process  $B_c^+ \rightarrow D^{(*)+} J/\psi \rightarrow D^0 \pi^+$  can be easily obtained from the integrals (13), (14) and (15). The dispersive part of  $B_c^+ \rightarrow D^{(*)+} J/\psi \rightarrow D^0 \pi^+$  can be related to the absorptive part via the dispersive relation [10, 21]

$$Dis[B_c^+ \rightarrow D^0 \pi^+] = \frac{1}{\pi} \int_{s_1}^{\infty} \frac{Abs[B_c^+ \rightarrow D^0 \pi^+]}{s - m_{B_c}^2} ds. \quad (18)$$

However, the cutoff which is phenomenologically introduced and the complicated integral in eq. (18) would cause unavoidable uncertainties to the dispersive part. In some of the former works, for estimating the decay width, the contribution of dispersive part was assumed to be small comparing with that of the absorptive part and ignored. However, for the direct CP violation, we must estimate the dispersive part and determine the strong phase induced by the triangle diagram, otherwise the strong phase would be exactly  $\pi/2$ .

In this work, adopting the method in our previous work [22], we obtain the dispersive part of  $B_c^+ \rightarrow D^{(*)+} J/\psi \rightarrow D^0 \pi^+$  by directly calculating the triangle where the intermediate hadrons are not on their mass shells. The amplitudes corresponding to the process of  $B_c^+ \rightarrow D^{*+}(p_1) J/\psi(p_2) \rightarrow \pi^+(p_3) D^0(p_4)$  where  $D^0$  or  $D^{*0}$  are exchanged are

$$\begin{aligned} Dis^{(b)} &= \int \frac{d^4 q}{(2\pi)^4} \mathcal{M}[B_c^+(p) \rightarrow D^{*+}(p_1) J/\psi(p_2)] [-g_{D^* D \pi}(-ip_3^\xi)] [-g_{\psi D D}(ip_4 - iq)^\mu] \\ &\quad \times (-g_{\sigma\xi})(-g_{\omega\mu}) \frac{i}{p_1^2 - m_D^2} \frac{i}{p_2 - m_{J/\psi}} \frac{i}{q^2 - m_D^2} \mathcal{F}^2[q^2, m_D^2], \end{aligned} \quad (19)$$

and

$$\begin{aligned} Dis^{(c)} &= \int \frac{d^4 q}{(2\pi)^4} \mathcal{M}[B_c^+(p) \rightarrow D^{*+}(p_1) J/\psi(p_2)] [-ig_{D^* D^* \pi} \varepsilon^{\mu\nu\alpha\beta}(-ip_{1\mu})(-iq_\alpha)] \\ &\quad [-ig_{J/\psi D^* D} \varepsilon^{\xi\lambda\kappa\rho}(-ip_{2\xi})iq_\kappa] (-g_{\beta\rho})(-g_{\sigma\nu})(-g_{\omega\lambda}) \frac{i}{p_1^2 - m_D^2} \frac{i}{p_2 - m_{J/\psi}} \\ &\quad \times \frac{i}{q^2 - m_{D^*}^2} \mathcal{F}^2[q^2, m_{D^*}^2]. \end{aligned} \quad (20)$$

Due to the existence of the dipole form factors  $\mathcal{F}^2[q^2, m_D^2]$  and  $\mathcal{F}^2[q^2, m_{D^*}^2]$  the ultraviolet behavior of the triangle loop integration is benign. These form factors play an equivalent role to the  $\Lambda$ -related terms introduced in the Pauli-Villas renormalization scheme [23, 24]. Because the final expressions of eqs. (19) and (20) are complicated, we would collect some useful formulas in appendix.

### C. Direct CP violation

The observable direct CP violation is defined as

$$\mathcal{A}_{CP} = \frac{|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\overline{\mathcal{M}}|^2} \quad (21)$$

with

$$\begin{aligned}\mathcal{M} &= \mathcal{M}^{dir}(B_c^+ \rightarrow D^0 \pi^+) + \mathcal{M}^{FSI}(B_c^+ \rightarrow D^0 \pi^+), \\ \overline{\mathcal{M}} &= \mathcal{M}^{dir}(B_c^- \rightarrow \bar{D}^0 \pi^-) + \mathcal{M}^{FSI}(B_c^- \rightarrow \bar{D}^0 \pi^-),\end{aligned}$$

where  $\mathcal{M}^{dir}(B_c^+ \rightarrow D^0 \pi^+)$  and  $\mathcal{M}^{dir}(B_c^- \rightarrow \bar{D}^0 \pi^-)$  were calculated in the approach of PQCD by the many authors [18] and they are written as

$$\mathcal{M}^{dir}(B_c^+ \rightarrow D^0 \pi^+) = V_u(T_u + P)[1 - ze^{i(-\gamma+\delta)}], \quad (22)$$

$$\mathcal{M}^{dir}(B_c^- \rightarrow \bar{D}^0 \pi^-) = V_u^*(T_u + P)[1 - ze^{i(\gamma+\delta)}] \quad (23)$$

with

$$z = \left| \frac{V_c}{V_u} \right| \left| \frac{T_c + P}{T_u + P} \right| \quad \text{and} \quad \delta = \arg \left[ \frac{T_c + P}{T_u + P} \right],$$

where  $V_u = V_{ud}V_{ub}^*$  are the Cabibbo-Kabayashi-Maskawa entries,  $|V_c/V_u| = \frac{\lambda}{1-\lambda^2/2}|V_{cb}/V_{ub}|$ . The values of  $T_{u,c}$ ,  $P$ ,  $\gamma$ ,  $\lambda$ ,  $z$  and  $\delta$  are given in Ref. [18], which are listed in Table I.

The amplitude of  $B_c^+ \rightarrow D^0 \pi^+$  induced by the FSI effect which is denoted by the subscript "FSI" is:

$$\mathcal{M}^{FSI}(B_c^+ \rightarrow D^0 \pi^+) = Dis + i \sum_{j=a,b,c} Abs^{(j)}, \quad (24)$$

and

$$\mathcal{M}^{FSI}(B_c^+ \rightarrow D^0 \pi^+) = \mathcal{M}^{FSI}(B_c^- \rightarrow \bar{D}^0 \pi^-). \quad (25)$$

$T_u$	$22.621 + 0.863i$	$\delta$	$123^\circ$
$T_c$	$-0.83 + 3.57i$	$\gamma$	$55^\circ$
$P$	$-0.474 - 1.722i$	$ \frac{V_{ub}}{V_{cb}} $	0.085
$z$	0.28		

TABLE I: These values are taken from Ref. [18]. Here  $T_{u,c}$  and  $P$  are in unit of  $10^{-3}$  GeV. In this work, we need multiply a factor  $\sqrt{m_{B_c}^5} G_F / \sqrt{2} |\mathbf{k}| \sim 4.83 \times 10^{-4}$  to  $T_{u,c}$  and  $P$ , because the formula for the decay widths adopted in this work takes a different normalization from that in Ref. [18].

### III. NUMERICAL RESULTS

The input parameter set which we are going to use in this work, includes:  $m_{B_c} = 6.286$  GeV,  $m_{J/\psi} = 3.097$  GeV,  $m_{D^+} = 1.869$  GeV,  $m_{D^{*+}} = 2.01$  GeV,  $m_{D^0} = 1.865$  GeV [25];  $f_\psi = 405 \pm 17$  MeV [25];  $f_D = 222.6 \pm 16.7_{-3.4}^{+2.8}$  MeV,  $f_{D^*} = 245 \pm 20_{-2}^{+3}$  MeV [26].  $V_{ud} = 0.974$ ,  $V_{cd} = 0.230$ ,  $V_{cb} = 0.0416$  [25].  $g_{D^*D\pi} = 17.3$ ,  $g_{D^*D^*\pi} = 8.9$  GeV $^{-1}$ ,  $g_{DD\psi} = 7.9$ ,  $g_{D^*D\psi} = 4.2$  GeV $^{-1}$  [27];  $a_1 = 1.14$ ,  $a_2 = -0.20$  [5].  $V_{ub} = A\lambda^3(\rho - i\eta) = 0.00218 - 0.00335i$ . The wolfenstein parameters of CKM matrix elements:  $\lambda = 0.2272$ ,  $A = 0.818$ ,  $\bar{\rho} = 0.221$  and  $\bar{\eta} = 0.340$  with  $\bar{\rho} = \rho(1 - \frac{\rho}{2})$  and  $\bar{\eta} = \eta(1 - \frac{\rho}{2})$ .  $G_F = 1.16637 \times 10^{-5}$  GeV $^{-2}$  [25].

The form factors in processes  $B_c \rightarrow D^{(*)}$  and  $B_c \rightarrow J/\psi$  possess pole structures [6, 7]

$$F(q^2) = \frac{F(0)}{1 - a\zeta + b\zeta^2} \quad (26)$$

with  $\zeta = q^2/m_{B_c}^2$ . The values of  $F(0)$ ,  $a$  and  $b$  are evaluated by some authors and for readers' convenience, we list their results in Table II.

		$f_+$	$f_-$	$F_+^A$	$F_-^A$	$F_0^A$	$F_V$
$D$	$F(0)$	0.189	-0.194	-	-	-	-
	$a$	2.47	2.43	-	-	-	-
	$b$	1.62	1.54	-	-	-	-
$D^*$	$F(0)$	-	-	0.158	-0.328	0.284	0.296
	$a$	-	-	2.15	2.40	1.30	2.40
	$b$	-	-	1.15	1.51	0.15	1.49
$J/\psi$	$F(0)$	-	-	0.66	-1.13	0.68	0.96
	$a$	-	-	1.13	1.23	0.59	1.24
	$b$	-	-	-0.067	0.006	-0.483	-0.002

TABLE II: The values of  $F(0)$ ,  $a$  and  $b$  in the form factors of  $B_c \rightarrow D^{(*)}$  and  $B_c \rightarrow J/\psi$  [6, 7].

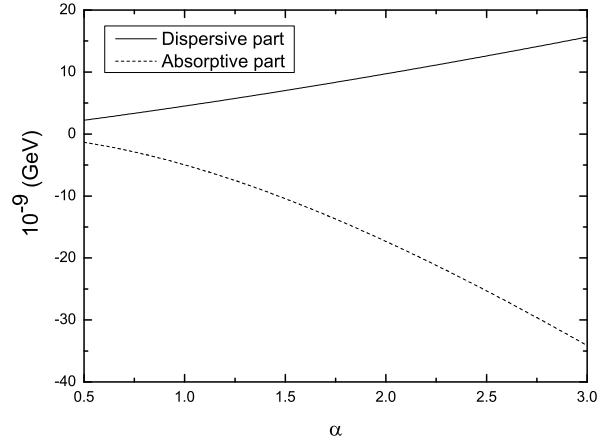


FIG. 2: The dispersive part and absorptive part of the amplitudes of  $B_c^+ \rightarrow D^{(*)+} J/\psi \rightarrow D^0 \pi^+$ .

In Fig. 2, we plot the dispersive part and absorptive part of the amplitudes of  $B_c^+ \rightarrow D^{(*)+} J/\psi \rightarrow D^0 \pi^+$  versus  $\alpha$  which is allowed to vary within  $\alpha = 0.5 \sim 3$ . In Fig. 3, we also list  $\mathcal{A}_{CP}$  with several typical values of  $\alpha$ . For a clear comparison, in this figure, we also give the value of  $\mathcal{A}_{CP}$  calculated in Ref. [18] by the PQCD approach which is purely induced by the short distance contribution (without considering the FSI). For clarity we also list some typical values of  $\mathcal{A}_{CP}$  with various  $\alpha$  in Table III.

#### IV. DISCUSSION AND CONCLUSION

Recently the direct CP violation in  $B$ -decays has been observed and it is expected to open a window for exploring new physics beyond the SM by which all theorists and experimentalists feel very inspired. Obviously, investigation of direct CP violation at  $B_c$  decays would be of special interests because it is composed of two heavy flavors and may be

$\alpha$	0.5	1.0	1.5	2.0	2.5	3.0
$\mathcal{A}_{CP}$	-30.2%	-28.1%	-25.3%	-22.4%	-19.8%	-17.5%

TABLE III: The typical values of  $\mathcal{A}_{CP}$



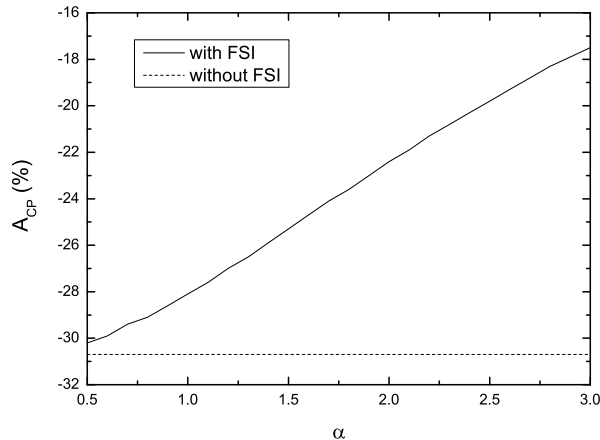


FIG. 3: The direct CP violation. Solid line and Dash line correspond to the direct CP with FSI effect and without FSI effect respectively.

more sensitive to new physics. On other aspect, before one can claim to find a trace of new physics, he must exhaust all possibilities in the framework of the SM. As indicated in literature, the FSI play an important role in B decays, therefore one has a full reason to expect that it is also significant at  $B_c$  decays. In this work, we carefully study the contribution of the FSI to the direct CP violation via its interference with the contribution from the short-distance effects which are induced by the tree and loop diagrams. Concretely, in this work, we calculate the amplitudes for  $B_c^+ \rightarrow D^0 \pi^+$  via sequent processes  $B_c^+ \rightarrow D^{(*)+} J/\psi \rightarrow D^0 \pi^+$  and determine its strong and weak phases.

Here we need to add some interpretation about application of PQCD. Even though we indicate the significance of the FSI for evaluating direct CP violation in Bc decays, their absolute contribution is much smaller than that from the direct process which is calculated in the framework of PQCD. Therefore if only the decay width of Bc is needed, one can ignore the contribution from the hadronic re-scattering, but as the CP violation is concerned as we see above, its contribution might be significant.

Our numerical results indicate that the typical value of  $A_{CP}$  with FSI effect is about  $-22\%$ , which is different from the value  $-30.7\%$  estimated in the PQCD approach without FSI [18]. On other aspect, one can also observe from Fig.3 that the effect of the hadronic re-scattering on  $A_{CP}$  may change quite diversely depending on the input parameter. Especially, as one adopts  $\alpha = 0.5$ ,  $A_{CP}$  is about  $-30.2\%$  which only slightly deviates from the value obtained in the framework of PQCD, however, as  $\alpha = 3$ , (even though  $\alpha = 3$  seems too large to be very reasonable, this effective coupling indeed can exceed 1 for hadron interaction, in fact, in some applications its value is set to be very large for fitting data)  $A_{CP}$  would change to  $-17.5\%$  obviously deviates from the value of PQCD. It indicates that the contribution of FSI to  $A_{CP}$  is of opposite sign with that from the quark loops and the cancellation may cause remarkable effects when one analyzes the data achieved in a rather precise measurement. Thus our conclusion is that the contribution from the FSI is not negligible.

In the future experiments, especially the LHCb, a great amount of data on  $B_c$  will be accumulated and one may have a possibility to measure the direct CP violation of  $B_c$ . If non-zero  $A_{CP}$  (almost definitely yes) is well measured, one can look for a trace of new physics by comparing the measured value with the theoretical result. When one compares the data which will be available at LHCb and/or other experiments with theoretical predictions, the contribution from the FSI must be included. This observation may be crucial for searching new physics in the future.

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## Appendix

Some useful formulas in the calculation of eqs. (19) and (20):

$$\begin{aligned}
& \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(p_1^2 - m_1^2)(p_2 - m_2)(q^2 - m^2)} \left( \frac{\Lambda^2 - m^2}{q^2 - \Lambda^2} \right)^4 \\
&= \frac{i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \left\{ \frac{(\Lambda^2 - m^2)y}{\Delta^2(m_1, m_2, \Lambda)} + \frac{1}{\Delta(m_1, m_2, \Lambda)} - \frac{1}{\Delta(m_1, m_2, m)} \right. \\
&\quad \left. - \frac{(-\Lambda^4 - m^4 + 2m^2\Lambda^2)y^2}{\Delta^3(m_1, m_2, \Lambda)} + \frac{(\Lambda^6 - 3m^2\Lambda^4 + 3m^4\Lambda^2 - m^6)y^3}{\Delta^4(m_1, m_2, \Lambda)} \right\}. \\
& \int \frac{d^4 q}{(2\pi)^4} \frac{l^2}{(p_1^2 - m_1^2)(p_2 - m_2)(q^2 - m^2)} \left( \frac{\Lambda^2 - m^2}{q^2 - \Lambda^2} \right)^4 \\
&= \frac{i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \left\{ \frac{-2y(\Lambda^2 - m^2)}{\Delta(m_1, m_2, \Lambda)} + 2 \ln \left[ \frac{\Delta(m_1, m_2, \Lambda)}{\Delta(m_1, m_2, m)} \right] \right. \\
&\quad \left. + \frac{(-\Lambda^4 - m^4 + 2m^2\Lambda^2)y^2}{\Delta^2(m_1, m_2, \Lambda)} - \frac{2y^3(\Lambda^6 - 3m^2\Lambda^4 + 3m^4\Lambda^2 - m^6)}{3\Delta^3(m_1, m_2, \Lambda)} \right\}. \\
& \int \frac{d^4 q}{(2\pi)^4} \frac{l^4}{(p_1^2 - m_1^2)(p_2 - m_2)(q^2 - m^2)} \left( \frac{\Lambda^2 - m^2}{q^2 - \Lambda^2} \right)^4 \\
&= \frac{i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \left\{ 6y(\Lambda^2 - m^2) \ln \left[ \frac{1}{\Delta(m_1, m_2, \Lambda)} \right] \right. \\
&\quad - 4y(\Lambda^2 - m^2) + 6 \left[ \Delta(m_1, m_2, \Lambda) \ln \left( \frac{1}{\Delta(m_1, m_2, \Lambda)} \right) \right. \\
&\quad \left. \left. - \Delta(m_1, m_2, m) \ln \left[ \frac{1}{\Delta(m_1, m_2, m)} \right] \right] - \frac{(3y^2\Lambda^4 - m^4 + 2m^2\Lambda^2)}{\Delta(m_1, m_2, \Lambda)} \right. \\
&\quad \left. + \frac{y^3(\Lambda^6 - 3m^2\Lambda^4 + 3m^4\Lambda^2 - m^6)}{\Delta^2(m_1, m_2, \Lambda)} \right\}.
\end{aligned}$$

Here

$$q = l - p_4 x + p_3(1 - x - y),$$

and

$$\begin{aligned}
\Delta(a, b, c) = & a^2(1 - x - y) + b^2x + c^2y + m_3^2(x^2 + y^2 - x - y + 2xy) + m_4^2(x^2 - x) \\
& + p_3 \cdot p_4(2x^2 + 2xy - 2x).
\end{aligned}$$

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